DETECTION OF ECOLOGICAL DISTURBANCES TO SEABED FAUNA THROUGH CHANGE OF WEIGHT DISTRIBUTION

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The effect of trawling on seabed fauna in the Northern Prawn Fishery experimental region of Australia is investigated through distributional changes in individual weights for each species. A stochastic growth model is employed to overcome a limited number of effective observations. One statistical challenge is to deal with non-identically distributed observations as only total weights and numbers of individuals caught for each species are observed. A modified Cramér-von Mises statistic is introduced and the p-values are evaluated by random number generation. As a result, the gamma distribution, the equilibrium distribution of the stochastic growth model fits well to 57 out of 80 cases before trawling. We conclude that most of the species are unaffected by trawling but several other species are shifted towards lighter weights. The unevenness of the effect over regions suggests that other environmental effects and ecological factors are involved.

Key words and phrases: Cramér-von Mises statistic, gamma distribution, stochastic growth model.

1. Introduction

Effects of various methods of harvesting the sea have been investigated in many articles on marine ecology. Collie *et al.* (2000) carried out a meta-analysis of 39 published fishing impact studies to draw general conclusions. Bishop *et al.* (2000) investigated the impact of technology on vessel performance in a trawl fishery during 1988–96 by using a generalized estimating equation. Burridge *et al.* (2003) investigated trawl-depletion rate for benthic fauna in an area closed to commercial trawling. In this paper we detect the effect of trawling through changes of weight distribution, the equilibrium distribution of a stochastic growth model. The stochastic growth model is frequently used for modelling population size (Russo *et al.* (2009)) or size of plants (Rupsys and Petrauskas (2007)) or animals (Tovar-Ávila *et al.* (2009)). It is shown that the equilibrium distribution of the model fits well as a reference distribution when no effective ecological disturbance exists. This result allows us to detect any effective disturbance by departure from the gamma distribution identified before the disturbance. The reason why we focus on individual weight as an index of disturbance in this paper is that it is sensitive to any ecological disturbances and easy to measure compared to size. An advantage of our approach is to be able to draw a whole

picture of the current status of each species on seabed before and after trawling without introducing any particular estimating equation or indices. The data used in this paper is that obtained in the project "Quantifying the effects of trawling on seabed fauna in the Northern Prawn Fishery" in Australia, which will be explained in detail in Section 2. Exhaustive analyses of the data were already reported in Haywood *et al.* (2005). After calculating various fundamental statistics and drawing many graphs and maps, they tried to find out the effect of trawling by application of a simple depletion and recovery model. Unfortunately it does not seem successful. There are several reasons why their analysis was not successful. One is that it is a class by class analysis, using popular descriptive statistics and plots. Our result shows that class by class analysis is too coarse to detect any effect of ecological disturbances. Instead we detect such disturbances by building up species by species stochastic growth models. Another reason is that their analysis is based on the whole weight of each species caught, normalised by the dredge area. The biomass density is a useful abundance measure for each survey area from the view point of fishery but not so for the detection of ecological disturbances. Changes of individual weight would be more useful for detecting ecological disturbance. Then, we detect it in this paper by fitting a stochastic growth model for individual weight. A technical problem which arises in analysing unnormalised weight with the number of catches of each species is that we have to deal with non identically distributed variables. This does not cause any serious problem in parameter estimation but requires a modification of the goodness of fit test statistic, most of which are proposed for i.i.d. (independent and identically distributed) observations. We evaluate p-values of a modified Cramér-von Mises statistic by executing computer simulations around the maximum likelihood estimate since even the asymptotic distribution is unknown. The stochastic growth model is introduced in Section 3 and a gamma distribution is derived as a stationary distribution for the weight. Distribution before trawling is identified in Section 4. The effect of trawling is detected through change of distribution in Section 5. Comparison with Welch's T test is given in Section 6.

2.   Seabed fauna in Northern Prawn Fishery and data

The data is from the Fisheries Research and Development Corporation (FRDC) funded Project 2002/102, "Quantifying the effects of trawling on seabed fauna in the Northern Prawn Fishery" (NPF) in Australia. The project was originally identified as a high priority research area by the Northern Prawn Fishery Management Advisory Committee (NORMAC) because under the Environmental Protection and Biodiversity Conservation Act (EPBC Act), Australian fisheries are required to demonstrate their environmental sustainability. Industry offered special funding to support the research and the FRDC was asked to manage the project. The Commonwealth Scientific and Industrial Research Organisation (CSIRO) agreed to carry out the work, develop the scope of the work and the experimental design and contribute to the funding.

Trawlers in the NPF tend to concentrate their fishing on areas of higher

Table 1. Part of the data.

Region	Plot	Treatment	Time	Scientific name	Count	Weight (g)
East	12	4	Before	*Retiflustra cornea*	1	0.25
East	12	4	Before	*Melaxinaea vitrea*	1	9.16
East	12	4	Before	Tubeworm OPNO 006	14	2.28
East	12	4	Before	Neritidae OPNO 142	0	0
East	12	4	Before	*Leucosia whitei*	1	1.49
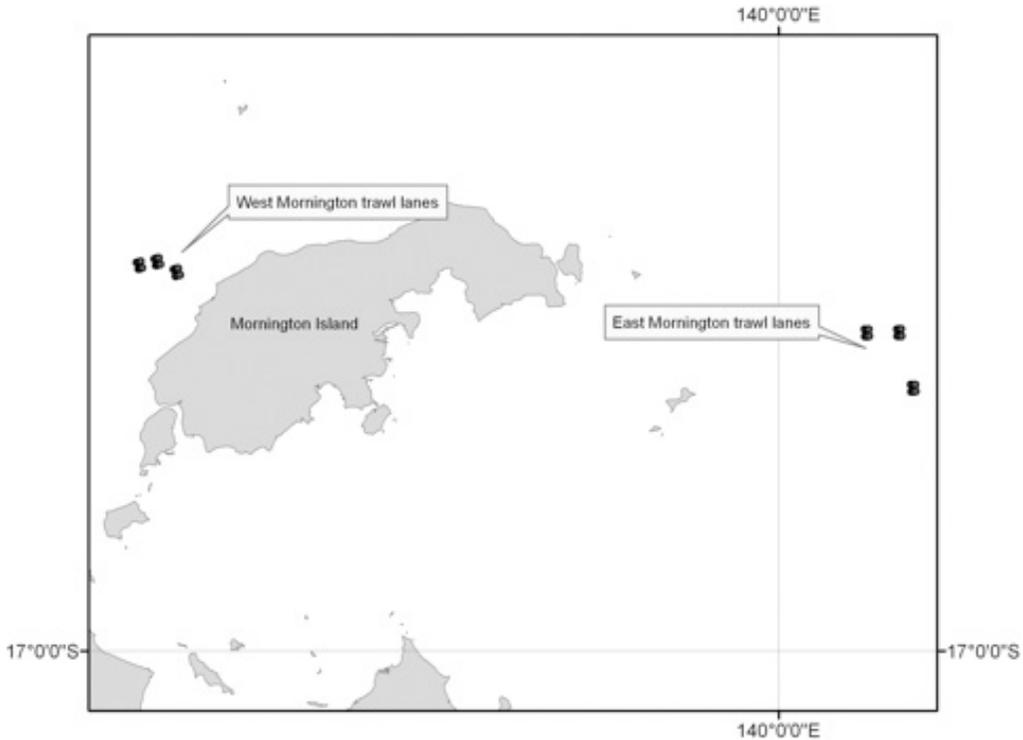
Figure 1. Two regions near Mornington Island used for the experimental survey (Haywood *et al.* (2005)).

prawn density. Also intensive trawling of small areas is a feature of the tiger and endeavour prawn fishery. Around 20% of the catch is prawns but the rest are animals collected from the seabed. Such by-catch animals can be seen in Table 7, although those are not caught by trawling but by experimental dredges.

Table 1 shows part of the data, which consists of 207,726 records, obtained by the experimental dredge survey for an explanation of the data structure. The first column labelled Region, indicates the region where the experiment was performed. The survey area shown in Fig. 1 is roughly divided into two regions, East (East of Mornington Island) and West (West of Mornington Island); three plots are set in each region, Plots 3, 5 and 6 in the West and Plots 9, 10, and 12 in

the East. Geographical features of the seabed of East and West are different; East is deeper but West is rougher and harder acoustically (Haywood *et al.* (2005)). Such a difference suggests the need of separate analyses for the East and West regions. At each plot, three levels of experimental trawlings (Treatments) were repeated three times. The three levels are the intensity of trawling, 0, 4, and 20, and the number of repeated trawlings on each plot. The trawled sea bed were dredged immediately after, or 6, 12 or 18 months after trawling as well as before trawling, indicated by the variable Time. Scientific Name is the name of the species caught by each dredge and Count is the number of individuals of each species caught in each trawl. Weight is the total weight of each case in grams.

Although the primary aim of the survey was to investigate the effect of different levels of trawl intensity and recovery time, we will concentrate on whether the effect of trawling is significant, since the number of effective observations is not large enough for a detailed analysis because of the large number of empty catches. Therefore, in this paper the treatment levels 4 or 20 are combined, and the weights recorded immediately after trawling are used for analysis in contrast with the weights before trawling. Although some useful information might be lost by combining those levels, it is a trade-off between the number of effective observations and the sharpness of the result. We may satisfy ourselves if the effect of trawling were detected in a systematic manner. Although 778 species were observed in this survey, only 16 classes of species were considered for analysis as there were too many zero catches for the other classes. Furthermore, 5 of the 16 classes are not appropriate for analysis. Demospongiae is hard to count since they are colonial, Pisces can easily escape from dredging and trawling, and Phaeophyta, Liliopsida and Chlorophyta are fragile plants difficult to collect intact. Eleven classes remained for our investigation: Hydrozoa, Anthozoa, Gymnolaemata, Polychaeta, Bivalvia, Gastropoda, Asteroidea, Ophiuroidea, Echinoidea, Crustacea and Ascidiacea. As a result 76 species remained in those classes for analysis, but we also removed those species with observations of less than 5, while the maximum number of observations for each case is 27 before trawling and 18 after trawling. We will report elsewhere other analyses of the data, for example, the number of individuals caught.

3. Weight distribution

3.1. Stochastic growth model and the equilibrium distribution

Richards (1969) shows that many of the deterministic growth models modify the relative growth rate $(1/X)dX/dt$,

$$\frac{dX_t}{X_t} = f(X_t)dt$$

where $f(X_t)$ is a function of the animal weight X_t at time t. An example of a deterministic model is the logistic,

$$(3.1) \qquad\qquad dX_t = \rho X_t(\kappa - X_t)dt$$

where κ is the growth limit and ρ is an exponential rate constant. It is one of the well known models for population growth (Davidson (1938), Smith (1963)), probably first proposed by Verhulst (1838). This model has since been used for describing many other aspects of growth other than that related to the population, for example, Marubini *et al.* (1972) analysed the growth of boys' and girls' heights.

We introduce a stochastic growth model for the individual's weight for each seabed fauna species at time t,

$$(3.2) \qquad dX_t = rX_t\left(1 - \frac{X_t}{k}\right)dt + \sigma X_t dB_t,$$

where r is the growth rate, k is the growth limit and B_t is a standard Brownian motion. Model (3.2) is a stochastic version of (3.1) but unlike (3.1) does not assume growth is monotonically increasing with time. In this respect the stochastic growth model (3.2) is a better model for weight. To explain the relation between these two models, the derivation by May (1973) is appealing. He assumed that the growth limit κ randomly fluctuates as $\kappa = k + \gamma(t)$ reflecting environmental changes for plants and animals. If it is written as

$$(\kappa - k)dt = \sigma_0 dB_t,$$

then (3.1) becomes

$$dX_t = (\rho k)X_t\left(1 - \frac{X_t}{k}\right)dt + (\rho\sigma_0)X_t dB_t,$$

which is equal to the model (3.2) when $r = \rho\kappa$ and $\sigma = \rho\sigma_0$. The source of random fluctuation in this derivation is that of the growth limit κ. In other words, individual difference comes from different values of κ in this model.

In this paper, we are interested in investigating the distributional change of the weight X_t rather than tracing the growth of individual weight. Let $p(x,t)$ be the probability density function of X_t. Then, as is shown in the Appendix, regardless of the shape of $p(x,t)$ it converges to the equilibrium distribution $p(x)$ as time t goes on, provided that $r > \sigma^2/2$ (p. 629, May (1973)). Note that the way of understanding a stochastic differential equation is a bit different from that of a non-stochastic differential equation. The random variable X_t describes the weight of individuals in the population at time t as a whole. Thus $p(t,x)$ is the distribution for whole individuals in the population at time t. Since the equilibrium distribution $p(x)$ is the distribution at the stage of equilibrium, $p(t,x)$ quickly goes back to $p(x)$ even if $p(x)$ were slightly modified by birth and death in the population. Of course, it takes longer to come back to the equilibrium if any big disturbance from outside occurs. Except for such a disturbed case, the equilibrium distribution can be used as a distribution of the weight, as far as X_t follows the stochastic differential equation (3.2). Therefore, it would be reasonable to use this equilibrium distribution $p(x)$ as a model for the weight of each species in a region where no disturbance from outside were made for an

adequate period as far as the individual weight increases or decreases along with the equation (3.2).

As has been suggested by May (1973), the equilibrium distribution of X_t is a gamma distribution $G_A(\nu, \alpha)$ with the shape parameter $\nu = 2r/\sigma^2 - 1$ and the scale parameter $\alpha = \sigma^2 k/2r$. As a reference, we give a simple proof for the convergence to the equilibrium distribution as time tends to infinity, with necessary conditions in the Appendix. The proof is similar to that of Goel and Richter-Dyn (1974), but the conditions are much simpler. It is worth noting that the solution does not remain the same if the definition of the stochastic integral is not an Ito type integral. The equilibrium distribution is not necessarily gamma if other integral types, such as the Stratanovich integral were employed (Feldman and Roughgarden (1975)).

It is of course possible to introduce another model for the weight. For example, a deterministic model

$$(3.3) \qquad\qquad dX_t = \rho \kappa X_t^{2/3} dt - \rho \kappa^{2/3} X_t dt$$

seems reasonable for the weight $X_t = Y_t^3$, where the length Y_t follows the von Bertalanffy model,

$$Y_t = Y_\infty - (Y_\infty - Y_0) \exp(-\beta t)$$

with $\rho = 3\beta/Y_\infty^2$, $\kappa = Y_\infty^3$. The differential equation (3.3) does a good job describing the difference between surface-proportional anabolism and weight-proportional catabolism (von Bertalanffy (1960)). We have tried to fit a stochastic modification of (3.3) to the given data,

$$(3.4) \qquad\qquad dX_t = \rho \kappa X_t^{2/3} dt - \rho \kappa^{2/3} X_t dt + \sigma X_t dB_t.$$

However, it did not work well. One of the reasons why it does not work well is the von Bertalanffy model is a deterministic model for the size, so that it is appropriate for the length of plant or any other increasing size but not for the weight which increases or decreases. In fact, the equilibrium distribution of the stochastic model (3.4) is a power transformed gamma distribution with the power $-1/3$. Such a negative power transformed gamma distribution does not seem reasonable. Also, the use of such a distribution for which no reproducibility property holds true causes a lot of problems in the estimation of parameters and test of goodness of fit. The distribution of the total weight of each case, which is the only available observation, becomes much more complicated. Although it is a big challenge to develop any other type of stochastic modification of the von Bertalanffy model, we concentrate ourselves on the model (3.2) in this paper.

3.2. Observed weight and maximum likelihood estimate

As has been mentioned, only total weights for each species in each dredge were recorded in this survey since individual weight measurement is time consuming and expensive. Let W_i represent the sum of the unobserved individual weights, X_j, so

$$W_i = X_1 + X_2 + \cdots + X_{m_i}, \qquad i = 1, \ldots, N,$$

where m_i is the number of individuals caught in Case i. Note that X_j's are not observable but W_i's are observed with m_i's. The observations $W_1, W_2, \ldots, W_N$ are now thought to be independent but not identically distributed. However, we can avoid other complications because of the reproducibility of the gamma distribution. That is, the W_i is still distributed as gamma $G_A(m_i\nu, \alpha)$, provided that X_j, $j = 1, 2, \ldots, m_i$, are independent and identically distributed as $G_A(\nu, \alpha)$, i.e. all individuals share the same scale α, and shape ν parameters. The maximum likelihood estimate of α is a function of ν,

$$\hat{\alpha} = \frac{w}{m\nu},$$

where $w = \sum_{i=1}^{N} w_i$ is the sum of observed total weights $w_1, w_2, \ldots, w_N$ and $m = \sum_{i=1}^{N} m_i$ is the sum of the number of individuals observed. Although no closed form is known for the maximum likelihood estimate of ν and α, we could obtain the numerical value of the estimate of ν by a numerical algorithm to maximise the profile likelihood,

$$L(\nu, \hat{\alpha}) = \sum_{i=1}^{N} \{-\log(\Gamma(m_i\nu)) + (m_i\nu - 1)\log w_i\} - m\nu \log\left(\frac{w}{m\nu}\right) - m\nu.$$

The function *nlminb* is an implementation of the nonlinear minimisation program developed at AT&T Bell Laboratories on R and available in any version of R as a part of standard library *stats*. The version of R we have used is 2.12.2, which is available from http://cran.r-project.org/.

3.3. Goodness of fit

Goodness of fit tests are indispensable when a model is fitted to data. One visual diagnostic tool is the quantile-quantile or Q-Q plot. However it is restricted to the case of independent and identically distributed observations. A similar visual diagnostic tool applicable to non-identically distributed observations would be a probability-probability or P-P plot (Gan and Koehler (1990), Holmgren (1995)) which is called the "Universal Q-Q plot" by Luceño (2007). The P-P plot for N independent observations $x_1, x_2, \ldots, x_N$, each of which is distributed as $F_k(x_k; \boldsymbol{\theta})$, $k = 1, 2, \ldots, N$, in our case which is gamma distribution with the parameter $(m_k\nu, \alpha)$ for the common parameter $\boldsymbol{\theta} = (\nu, \alpha)$, is a plot of the N points,

$$\left(\frac{k - 0.5}{N}, F_{(k)}(x_{(k)}; \boldsymbol{\theta})\right), \quad k = 1, 2, \ldots, N,$$

where $x_{(1)}, x_{(2)}, \ldots, x_{(N)}$ are order statistics of the observations and the distributions $F_k(x_k, \boldsymbol{\theta})$, $k = 1, 2, \ldots, N$, are arranged in the same order. The estimate $\hat{\boldsymbol{\theta}}$ is plugged into $\boldsymbol{\theta}$ if the parameter is unknown. A numerical goodness of fit statistic parallel to the P-P plot is then

$$\tilde{W}_N^2(\boldsymbol{\theta}) = \frac{1}{12N} + \sum_{k=1}^{N}\left(F_{(k)}(x_{(k)}\boldsymbol{\theta}) - \frac{k - 0.5}{N}\right)^2,$$

where the empirical estimate, $\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$, is used if the parameters are unknown as the same as in the P-P plot. This statistic is closely related to the Cramér-von Mises statistic,

$$W_N^2(\boldsymbol{\theta}) = N \int_{-\infty}^{\infty} (F_N(x) - F(x;\boldsymbol{\theta}))^2 dF(x;\boldsymbol{\theta})$$

$$= \frac{1}{12N} + \sum_{k=1}^{N} \left(F(x_{(k)};\boldsymbol{\theta}) - \frac{k-0.5}{N} \right)^2,$$

where $F_N(x)$ is the empirical distribution function and reduces to $\tilde{W}_N^2(\boldsymbol{\theta})$ when the observations are independent and identically distributed as $F(x;\boldsymbol{\theta})$. There are many articles on the behaviour of $W_N^2(\hat{\boldsymbol{\theta}})$ (Sukhatme (1972), Lockhart and Stephens (1985)) as well as that of $\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$ (Pierce and Kopecky (1979), Loynes (1980)). However, their results are not directly applicable for obtaining the p-value of $\tilde{W}_N^2(\boldsymbol{\theta})$ or $\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$ in our case, so that we will evaluate the p-value of $\tilde{W}_N^2(\boldsymbol{\theta})$ or $\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$ for the given observation based on 500 sets of generated random numbers which are distributed as $G_A(m_i\nu, \alpha)$, $i = 1, 2, \ldots, N$. The evaluation is not only at the point estimate $(m_i\hat{\nu}, \hat{\alpha})$ but also at several lattice points in the neighbourhood, $(m_i\nu, \alpha)$ for $\nu = 0.5\hat{\nu}$, $0.75\hat{\nu}$, $1.25\hat{\nu}$ and $1.5\hat{\nu}$ and $\alpha = 0.5\hat{\alpha}$, $0.75\hat{\alpha}$, $1.25\hat{\alpha}$ and $1.5\hat{\alpha}$. Table 2 shows one of the examples of the p-values in the neighbourhood. The data is for the case of Case 2 in the table in the Appendix. As is easily seen, the p-value does not fluctuate so much, ranging from 0.091 to 0.133, so that, we use the minimum in the neighbourhood as a p-value through this paper, which is favourable to the rejection of the fit.

Table 2. p-values of the goodness of fit statistic for Case 2.

	$0.5\hat{\alpha}$	$0.75\hat{\alpha}$	$\hat{\alpha}$	$1.25\hat{\alpha}$	$1.5\hat{\alpha}$
$0.5\hat{\nu}$	0.119	0.106	0.108	0.121	0.096
$0.75\hat{\nu}$	0.125	0.127	0.126	0.114	0.114
$\hat{\nu}$	0.129	0.128	0.125	0.108	0.133
$1.25\hat{\nu}$	0.115	0.106	0.114	0.118	0.122
$1.5\hat{\nu}$	0.115	0.091	0.099	0.103	0.092

4. Distribution before trawling

A gamma distribution is fitted to each observation before trawling. The results for all species are shown in Table 7 in the Appendix, where the gamma distribution $G_A(\nu, \alpha)$ is fitted to individual weights for the cases numbered from 1 to 80. Each case can be identified by a combination of its scientific name and the region name of the experiment. The class and family names are also listed in the table as a reference. Species identified by scientific name are grouped into a family and several families are further grouped into a class. We can see what kind of animals are by-catch from the seabed. N is the number of nonzero observations out of 27 observations in each case. The maximum likelihood estimates of the

parameters $\hat{\nu}$ and $\hat{\alpha}$ are also listed. The last two columns give the value of goodness of fit test statistic, $\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$, and the corresponding p-value. p-values less than 0.1 are marked by an asterisk in the table as a reference. It seems reasonable to exclude these cases which were not a good fit. 23 cases out of 80 cases were excluded for later analysis since we want to detect the effect of trawling through change of the equilibrium distribution of the stochastic growth model (3.2). To assist in the visual understanding of the goodness of fit of the 57 unmarked cases, several examples of P-P plots are given in Fig. 2 for Cases 1, 2 and 3.
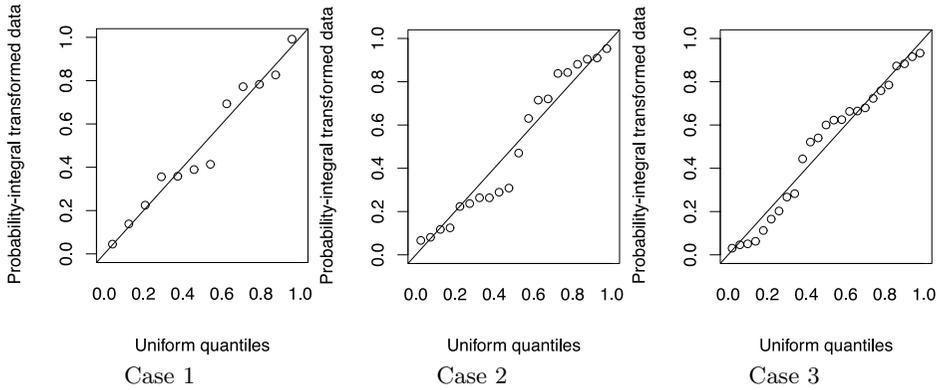
Figure 2. Examples of P-P plots for Case 1, Case 2 and Case 3.

To understand the meaning of the estimated parameters for the 57 good fit cases, a reasonable transform of the parameters would be $k = \alpha(\nu + 1)$ and $\xi = \sqrt{2/(\nu + 1)}$, because it is equivalent to rewrite the model (3.2) as

$$dX_s = X_s \left(1 - \frac{X_s}{k}\right) ds + \xi X_s dB_s,$$

where time is changed from t to $s = rt$. The parameter k is now the (deterministic) limit of growth and ξ is the size of random fluctuations around the limit k. Figure 3 is a scatter plot of $\hat{\xi} = \sqrt{2/(\hat{\nu} + 1)}$ and $\hat{k} = \hat{\alpha}(\hat{\nu} + 1)$ where Case 12 is excluded because $\hat{k} = 735.870$ is very large with $\hat{\xi} = 1.219$. The points on the scatter plot are identified by initial letters of class names. For example, H is the class Hydrozoa as is described on the legend. It is easily seen that the value of k is very large for 4 species, but it only implies that these species have a heavy dry weight. However, it is interesting to note that several classes share similar ξ values. It is less than 0.8 for Bivalvia, greater than 1.0 for Hydrozoa and between 0.7 and 1.1 for Gymnolaemata.

The stochastic growth model (3.2) may not be a good model for some of the remaining 23 cases. In fact, only one species *Dardanus imbricatus* shows a good fit both in the West and East out of four species in the list for which enough observations are available both in the West and East (Cases 6, 7, 41, 42, 50, 51, 57
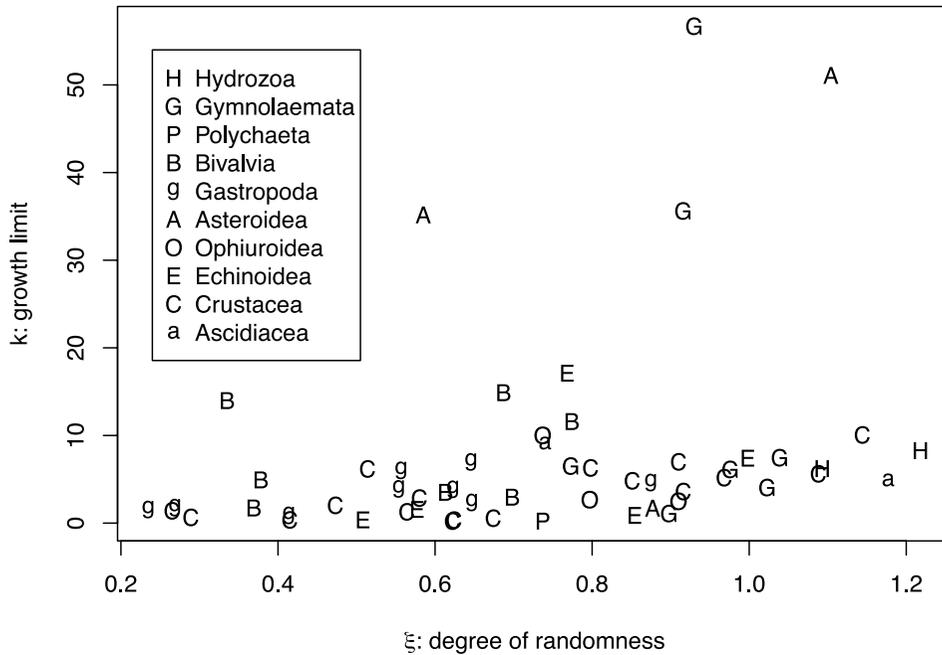
Figure 3. Scatter plot of the size of randomness $\hat{\xi} = \sqrt{2/(\hat{\nu}+1)}$ and the growth limit $\hat{k} = \hat{\alpha}(\hat{\nu}+1)$.

and 58). Possibly, in these cases, environmental factors may have delayed species maturity or the model (3.2) does not match to those 23 cases. Also, it is worth noting that there is no consistent rejection of the goodness of fit over different species. Therefore, we are forced to consider the species by species or case by case analysis. We leave any further understanding of the results, including a biological understanding, for future investigation.

5.   Effect of trawling

In this section we detect the effect of trawling through discrepancies in the identified gamma distribution rather than looking at gamma parameter changes. Although the discrepancy does not necessarily imply the existence of the effect of trawling, we note that the data is obtained under a careful design of experiments (Haywood *et al.* (2005)). Since there is no other effect than trawling we could say that we have detected the effect of trawling if the discrepancy is significant.

The statistical framework for the calculation of the *p*-value is different from that described in the previous section as no parameter estimation is involved in this stage. We calculate the *p*-value for the null distribution which is already identified before trawling. In other words, conditional *p*-values are calculated. We have simply obtained the *p*-value by generating 500 sets of random numbers distributed as the identified gamma distribution. The number of target cases now reduces to 50 since not enough individuals are available after trawling for the remaining cases. A summary of the result for the 50 cases is given in Table 3,

Table 3. Effect of trawling (U: Uneffected, L: Lighter, C: Changed).

Case	N	$\tilde{W}_N^2(\boldsymbol{\theta}_0)$	p-value	Effect	Case	N	$\tilde{W}_N^2(\boldsymbol{\theta}_0)$	p-value	Effect
1	6	0.092	0.636	U	34	9	0.115	0.550	U
3	15	0.106	0.584	U	36	14	0.091	0.650	U
5	9	0.076	0.734	U	40	7	0.319	0.106	U
7	13	0.059	0.838	U	41	6	0.319	0.112	U
9	8	0.077	0.748	U	45	7	0.096	0.650	U
11	8	0.144	0.424	U	46	18	0.264	0.184	U
12	6	0.097	0.604	U	47	11	0.143	0.404	U
13	17	1.075	0.000	L	49	9	0.658	0.022	L
14	8	0.223	0.212	U	51	13	0.166	0.348	U
15	12	0.441	0.058	L	52	7	0.367	0.088	L
19	9	0.150	0.422	U	53	12	1.556	0.000	C(L)
20	10	0.044	0.918	U	57	11	0.211	0.248	U
21	13	0.549	0.030	C(L)	58	8	0.191	0.276	U
22	12	0.183	0.334	U	59	14	0.242	0.210	U
23	10	0.446	0.054	L	60	7	0.097	0.640	U
24	12	0.125	0.530	U	61	12	0.390	0.094	C
25	9	0.212	0.272	U	63	10	0.200	0.250	U
26	15	0.458	0.036	C	70	7	0.066	0.808	U
27	14	0.184	0.322	U	71	15	0.188	0.300	U
28	8	0.225	0.208	U	73	6	0.277	0.136	U
30	9	0.122	0.516	U	75	10	0.697	0.008	C(L)
31	11	0.197	0.276	U	76	6	0.327	0.100	U
32	18	0.253	0.210	U	78	12	0.355	0.118	U
33	8	1.663	0.000	L					

where the uneffected cases are denoted by U, lighter cases by L and C denotes cases that change, but not consistently lighter nor heavier, in the column labelled Effect. There are no heavier cases in our study. The type of effect is determined by stochastic order. It is L if the discrepancy is significant with level 0.1 in terms of the p-value and $F_{(k)}(x_{(k)}; \boldsymbol{\theta_0}) < (k - 0.5)/N$ for $k = 1, \ldots, N$. The latter is equivalent if all points are below the 45 degree line on the P-P plot and C(L) where it is almost L, with a few exceptional points on the P-P plot. It is worth noting that almost all species are different except Cases 57 and 58, suggesting that the effect of trawling on a species weight is different for different regions East or West.

We can see more detail about changes of the distribution through P-P plots. Figure 4 gives us such plots for 6 cases of type L. A possible interpretation for type L species would be that those species have difficulty avoiding the trawl net and only individuals smaller than the net size remain, so that the distribution is skewed in the lighter direction.

Figure 5 gives us the plots for 5 cases of type C or C(L). It is easily understood that the distribution is skewed to lighter weights for Case 21, Case 53
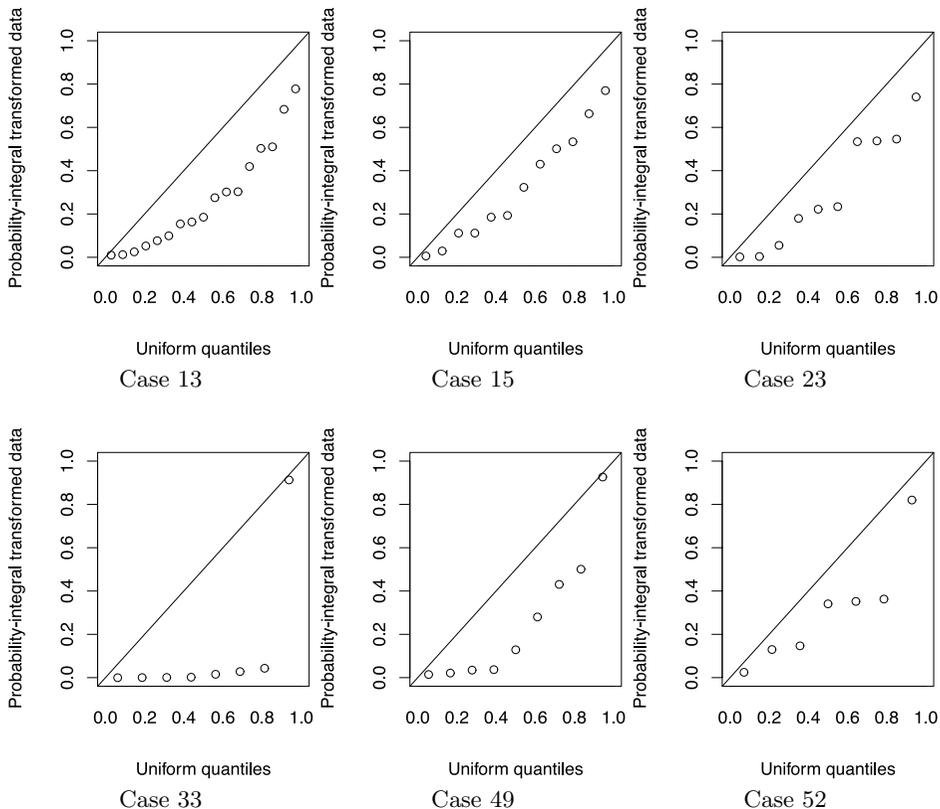
Figure 4. P-P plots for six cases where the weight distribution became lighter after trawling. The null distributions are $G_A(1.313, 24.512)$, $G_A(2.687, 0.062)$, $G_A(4.320, 0.660)$, $G_A(35.112, 0.043)$, $G_A(2.388, 5.049)$ and $G_A(5.018, 0.259)$, respectively.

and Case 75, if a single point on the P-P plot was neglected. However, there is no clear direction of change for the other cases, Case 26 and Case 61. This is probably because such species are more sensitive to other factors like the local unevenness of the environment rather than that of trawling.

6. Comparison with simple mean tests

We have seen the effectiveness of the gamma distribution in detecting the effect of trawling. It is however worthwhile to compare this result with that obtained by a simple mean difference test statistic like Welch's T as

$$T = \frac{\bar{W}_a - \bar{W}_b}{\sqrt{\dfrac{s_a^2}{N_a} + \dfrac{s_b^2}{N_b}}},$$

where $\bar{W}_a$ is the sample mean of weights normalised by the number of individuals caught after trawling, s_a^2 is the sample variance and N_a is the sample size, and $\bar{W}_b$, s_b^2 and N_b are those for the normalised weights before trawling. Table 4
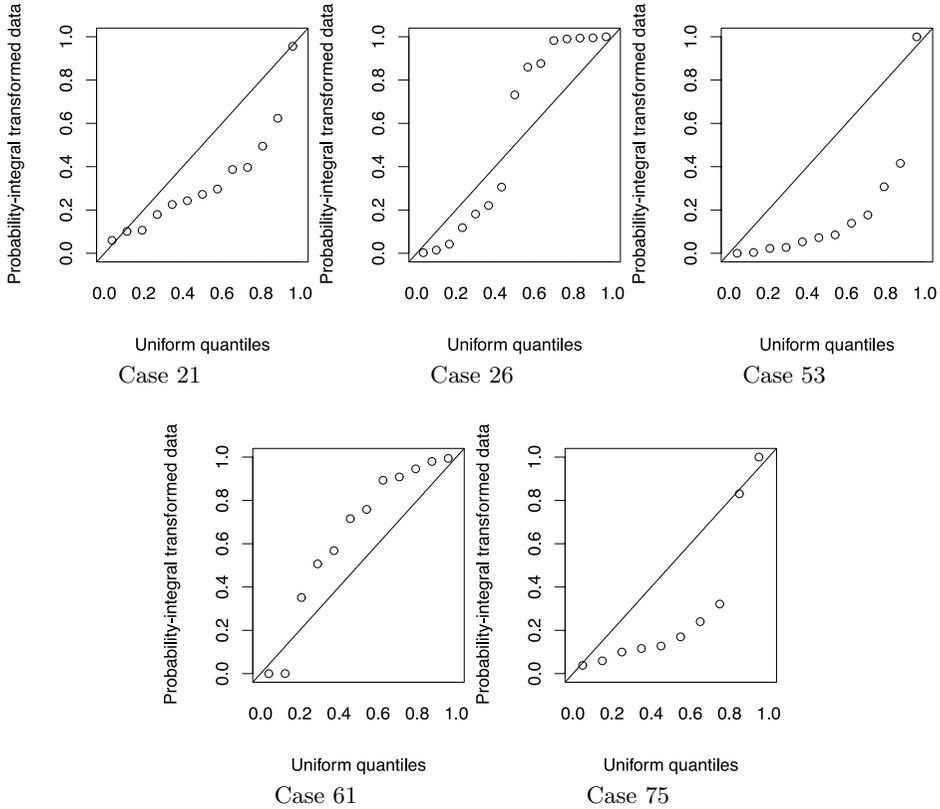
Figure 5. P-P plots for five cases where the weight distribution changed without direction after trawling. The null distributions are $G_A(2.342, 3.467)$, $G_A(26.664, 0.0463)$, $G_A(1.007, 3.693)$, $G_A(27.462, 0.049)$ and $G_A(0.690, 3.327)$, respectively.

shows the p-values for our statistic and for the T statistic for the two sided alternative hypothesis in the case of type U. Also p-values for the Student's T test statistic,

$$T = \frac{\bar{W}_a - \bar{W}_b}{\sqrt{\dfrac{s_a{}^2}{N_a}}}$$

given $\bar{W}_b$, are shown in the table for reference, since the p-values for $\tilde{W}_N^2(\boldsymbol{\theta_0})$ are calculated conditionally on the data before trawling.

It seems reasonable that the p-values for Welch's T are all large for the cases of type U. However, the values themselves are not consistent with those for $\tilde{W}_N^2(\boldsymbol{\theta_0})$, particularly for the three cases marked † in the table. The reason is that the discrepancy from the null distribution is symmetric, so that the mean difference fails in describing such a discrepancy as is seen from the P-P plots given in Fig. 6. The p-values for the Student's T are not consistent with those for Welch's T, particularly for the three cases marked ∗ in this table. The reason is that the conditional test is sensitive to a slight shift of the distribution as is

Table 4. Comparison of the p-values for type U.

Case	1	3	5	7	9	11	12
p-value ($\tilde{W}_N^2(\boldsymbol{\theta_0})$)	0.636	0.584	0.734	0.838	0.748	0.424	0.604
p-value (Welch's T)	0.524	0.541	0.735	0.642	0.376	0.756	0.804
p-value (Student's T)	0.315	0.428	0.690	0.507	0.354	0.734	0.679
$\mathrm{sign}(T)$	$-$	$-$	$+$	$-$	$+$	$+$	$-$

14	19	20	22	24	25	27	28	30	31*
0.212	0.422	0.918	0.334	0.530	0.272	0.322	0.208	0.516	0.276
0.318	0.777	0.975	0.757	0.810	0.465	0.423	0.628	0.674	0.154
0.130	0.756	0.960	0.592	0.771	0.135	0.298	0.269	0.526	0.097
$-$	$-$	$-$	$+$	$-$	$+$	$-$	$+$	$+$	$+$

32	34	36	40	41	45	46†	47	51	57*
0.210	0.550	0.650	0.106	0.112	0.650	0.184	0.404	0.348	0.248
0.851	0.967	0.721	0.177	0.260	0.981	0.992	0.954	0.393	0.157
0.808	0.955	0.609	0.105	0.186	0.976	0.990	0.947	0.279	0.016
$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$-$	$-$

58	59*	60	63	70	71	73	76†	78†
0.276	0.210	0.640	0.250	0.808	0.300	0.136	0.100	0.118
0.372	0.269	0.963	0.766	0.730	0.425	0.338	0.606	0.833
0.330	0.055	0.945	0.682	0.670	0.354	0.301	0.595	0.797
$-$	$-$	$+$	$+$	$-$	$+$	$-$	$+$	$-$
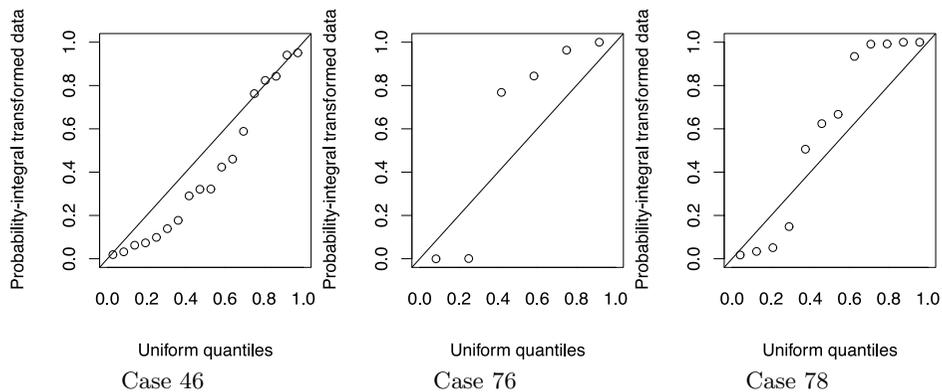
Figure 6. P-P plots for Case 46, 76 and 78 after trawling.

seen from P-P plots for those three cases given in Fig. 7.

Table 5 is a similar table for type L. It is clear that Welch's T fails in detecting changes in the 3 cases marked by $\dagger$, although the values of Welch's T are all negative. The sensitivity of Student's T makes a difference for Case 23 as well as for Type U.

Table 6 is for Type C or C(L). In this case, Welch's T fails in describing
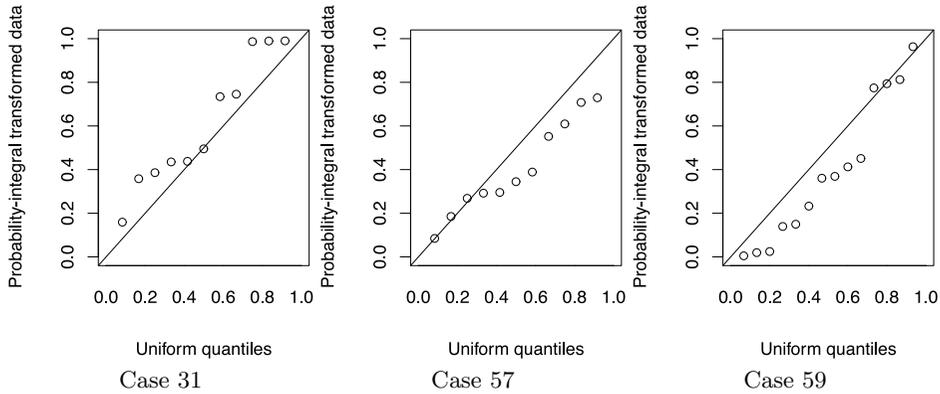
Figure 7. P-P plots for Case 31, 57 and 59 after trawling.

Table 5. Comparison of the p-values for Type L.

Case	13	$15^\dagger$	$23^\dagger$	33	49	$52^\dagger$
p-value $(\tilde{W}_N^2(\boldsymbol{\theta_0}))$	0.000	0.058	0.054	0.000	0.022	0.088
p-value (Welch's T)	0.005	0.291	0.194	0.010	0.020	0.486
p-value (Student's T)	0.000	0.174	0.014	0.009	0.013	0.384
sign(T)	$-$	$-$	$-$	$-$	$-$	$-$

Table 6. Comparison of the p-values for Type C or C(L).

Case	21	$26^\dagger$	53	$61^\dagger$	$75^\dagger$
p-value $(\tilde{W}_N^2(\boldsymbol{\theta_0}))$	0.030	0.036	0.000	0.094	0.008
p-value (Welch's T)	0.084	0.168	0.056	0.937	0.720
p-value (Student's T)	0.020	0.073	0.008	0.928	0.696
sign(T)	$-$	$+$	$-$	$-$	$+$

changes with a level of 0.1 in 3 out of the 5 cases. A significant difference is shown for Case 75, where the sign of Welch's T is positive although it belongs to type C(L). Similar comments apply for the Student's T as these for Type L. As a summary, Welch's T tends to fail in the detection of distributional changes when the empirical distribution differs from the null distribution in a symmetric manner. On the other hand, Student's T seems very sensitive for slight differences from the null distribution. Such mean difference tests are simple and easy to use, but not strong enough for detection of distributional changes since the distributions are only identified by the mean.

7. Discussion

We have shown that the gamma distribution, the equilibrium distribution of the stochastic growth model, not only describes the distribution of species weights but also provides good support to detect the effect of trawling for each species,

including the direction of changes. Goodness of fit of the distribution is tested by a modified Cramér Von Mises type statistic with a graphical representation of the fit by a P-P plot. One of the reasons why we need such a test is that only the total weights of catches for each species are recorded in this survey. As a result the integrated use of numerical and graphical tests of goodness of fit provide us with a better understanding of the trawling effect. It is obvious that formal testing of gamma parameter changes does not lead us to a complete understanding of distributional changes. It is always a problem in biological data analysis to be able to cope with small numbers of effective samples. As has been shown in this paper, the introduction of a descriptive model like (3.2) can boost our analysis but it is not enough. Augmentation of data, for example, combining different treatment levels as we have done, is also needed to cope with a limited number of observations. It is of course at the cost of losing some important aspects of the data. In fact, the augmentation destroyed part of the design of experiments planned before the survey. A suggestion would then be that simple but more frequent experimental dredges are desirable for detecting the effect of trawling in future surveys.

A limitation of our analysis is that nothing can be said about the 23 species to which the equilibrium distribution does not fit well. It is a further challenge to develop another model which is applicable to these remaining species. We could classify the remaining 50 species into three types, unaffected, lighter and changed in Section 5. The fact that no heavier species were found is an interesting result which supports our analysis. However, it is unfortunate that nothing more can be said about details of the effect, for example, region specific environmental effects or ecological factors. We hope that our results will be helpful for further understanding of the effect as well as making a new sample survey.

A further challenge is to have a deeper biological understanding of the results. For example, we have discussed with biologists many times how to interpret the pair of the values of ξ and k, which are displayed on the plot in Fig. 3. However, it was not easy for them because most of the by-catch species are new to them. We hope that our results including Table 7 will help deepen the understanding of seabed fauna in the Northern Prawn Fishery, not only from the view point of fishery, but also the view point of ecology.

Appendix
Equilibrium Distribution of the Growth Model

In this appendix, the equilibrium distribution is derived in a general frame work when $\{X_t\}$ satisfies

$$dX_t = a(X_t)dt + b(X_t)dB_t.$$

It is well known that the probability density function of $p(t,x)$ for X_t satisfies the Kolmogorov's forward equation,

$$\frac{\partial p(t,x)}{\partial t} = -\frac{\partial}{\partial x}(a(x)p(t,x)) + \frac{1}{2}\frac{\partial^2}{\partial x^2}(b^2(x)p(t,x)).$$

The equilibrium distribution $p(x) = \lim_{t \to \infty} p(t, x)$, then satisfies the equation

(A.1) $$0 = -\frac{d}{dx}(a(x)p(x)) + \frac{1}{2}\frac{d^2}{dx^2}(b^2(x)p(x)).$$

By integrating the both sides of (A.1), we have

$$a(x)p(x) - \frac{1}{2}\frac{d}{dx}(p(x)b^2(x)) + C = 0,$$

for a constant C, which is not necessarily null. However, C becomes 0 as far as $p(x)$ satisfies the following boundary conditions

$$\lim_{x \to \infty} p(x)a(x) = 0 \quad \text{and} \quad \lim_{x \to \infty} \frac{d}{dx}(p(x)b^2(x)) = 0.$$

Then $q(x) = p(x)b^2(x)$ is the solution of

$$\frac{d}{dx}q(x) - \frac{2a(x)}{b^2(x)}q(x) = 0,$$

that is

$$p(x) = \frac{C'}{b^2(x)}\exp\left(\int \frac{2a(u)}{b^2(u)}du\right),$$

where C' is a constant.

In case of our growth model, since $a(x) = rx(1 - x/k)$ and $b(x) = \sigma x$, we have the gamma distribution

$$\begin{aligned} p(x) &= \frac{C'}{(\sigma x)^2}\exp\left(\int \frac{2rx(1 - x/k)}{(\sigma x)^2}dx\right) \\ &= \frac{C}{\sigma^2}x^{-2}\exp\left(\frac{2r}{\sigma^2}\int\left(\frac{1}{x} - \frac{1}{k}\right)dx\right) \\ &= \frac{1}{\alpha^\nu\Gamma(\nu)}x^{\nu-1}\exp\left(-\frac{x}{\alpha}\right) \end{aligned}$$

for $\nu = \frac{2r}{\sigma^2} - 1$ and $\alpha = \frac{k}{\nu+1}$.

Acknowledgements

Table 7. Goodness of fit of gamma distribution before trawling.

Case	Class	Family	Scientific name	Region	N	shape $\hat{\nu}$	scale $\hat{\alpha}$	$\tilde{W}_N^2(\boldsymbol{\theta})$	p-value
1	Hydrozoa		Hydroid OPNO 006	East	12	0.674	3.712	0.045	0.653
2	Hydrozoa		Hydroid OPNO 156	West	20	1.140	0.967	0.111	0.091*
3	Hydrozoa		Hydroid OPNO 184	East	25	0.349	6.127	0.075	0.409
4	Hydrozoa		Hydroid OPNO 201	East	6	1.190	1.366	0.125	0.042*
5	Gymnolaemata	Flustridae	*Retiflustra cornea*	East	18	1.480	0.439	0.067	0.335
6	Gymnolaemata		Cheilostomata sp OPNO 142	West	7	1.024	1.168	0.120	0.059*
7	Gymnolaemata		Cheilostomata sp OPNO 142	East	17	2.344	1.947	0.102	0.107
8	Gymnolaemata		Scrupocellaria sp OPNO 215	East	7	0.912	2.124	0.047	0.610
9	Gymnolaemata		Bryozoan OPNO 142a	West	11	0.852	4.030	0.035	0.793
10	Gymnolaemata		Bryozoan OPNO 142b	West	21	0.686	13.502	0.163	0.021*
11	Gymnolaemata		Bryozoan OPNO 171a	East	16	1.098	2.918	0.102	0.130
12	Gymnolaemata		Bryozoan OPNO 171b	East	16	0.346	546.803	0.100	0.174
13	Gymnolaemata		Bryozoan OPNO 203	East	21	1.313	24.512	0.068	0.330
14	Gymnolaemata		Bryozoan OPNO 216	East	9	1.382	14.952	0.050	0.519
15	Polychaeta		Tubeworm OPNO 006	East	21	2.687	0.062	0.116	0.144
16	Bivalvia	Nuculidae	*Leionucula superba*	East	6	3.102	0.721	0.060	0.420
17	Bivalvia	Glycymerididae	*Melaxinaea vitrea*	East	14	2.560	3.187	0.397	0.000*
18	Bivalvia	Malleidae	*Malleus (Malleus) malleus*	East	25	1.233	6.478	0.163	0.036*
19	Bivalvia	Pectinidae	*Amusium pleuronectes*	East	9	16.855	0.783	0.079	0.231
20	Bivalvia	Pectinidae	*Annachlamys flabellata*	East	13	3.235	3.514	0.052	0.551
21	Bivalvia	Spondylidae	Spondylidae OPNO 193	East	18	2.342	3.467	0.056	0.540
22	Bivalvia	Cardiidae	Cardiidae OPNO 151	West	9	12.990	0.353	0.026	0.917
23	Bivalvia	Veneridae	Lioconcha sp OPNO 004	East	7	4.320	0.660	0.031	0.895
24	Bivalvia	Veneridae	Placamen sp OPNO 156	West	10	13.680	0.117	0.067	0.355
25	Gastropoda	Neritidae	Neritidae OPNO 142	West	10	3.780	0.496	0.067	0.326
26	Gastropoda	Modulidae	Modulidae OPNO 151	West	9	26.664	0.064	0.038	0.726

Table 7. (continued).

Case	Class	Family	Scientific name	Region	N	shape $\hat{\nu}$	scale $\hat{\alpha}$	$\tilde{W}_N^2(\hat{\boldsymbol{\theta}})$	p-value
27	Gastropoda	Strombidae	Strombus sp OPNO 142	West	13	5.442	0.936	0.035	0.815
28	Gastropoda	Strombidae	Strombus sp OPNO 150	West	9	4.147	0.752	0.051	0.579
29	Gastropoda	Muricidae	Chicoreus sp OPNO 184	East	24	2.449	1.211	0.130	0.057*
30	Gastropoda	Muricidae	Murex sp OPNO 002	East	8	1.613	1.775	0.070	0.286
31	Gastropoda	Muricidae	Murex sp OPNO 172	East	9	5.524	0.600	0.029	0.889
32	Gastropoda	Cerithiidae	Cerithiidae OPNO 142	West	21	10.651	0.077	0.100	0.182
33	Gastropoda	Cancellariidae	Cancellariidae OPNO 151	West	10	35.112	0.043	0.050	0.514
34	Gastropoda	Architectonicidae	Architectonica sp OPNO 151	West	8	3.794	1.459	0.061	0.396
35	Gastropoda	Smaragdinellidae	Smaragdinellidae OPNO 151	West	15	1.215	0.779	0.584	0.000*
36	Asteroidea	Luidiidae	Luidiidae OPNO 006	East	22	1.603	0.655	0.038	0.771
37	Asteroidea	Astropectinidae	Astropectinidae OPNO 006	East	18	2.804	0.726	0.131	0.055*
38	Asteroidea	Astropectinidae	Astropectinidae OPNO 142	West	26	0.319	4.116	0.287	0.000*
39	Asteroidea	Goniasteridae	Stellaster sp OPNO 006a	East	9	0.492	132.040	0.124	0.070*
40	Asteroidea	Goniasteridae	Stellaster sp OPNO 006b	East	9	4.852	6.010	0.073	0.325
41	Asteroidea	Goniasteridae	Stellaster sp OPNO 118	West	6	0.641	31.123	0.087	0.186
42	Asteroidea	Goniasteridae	Stellaster sp OPNO 118	East	22	0.468	14.756	0.142	0.043*
43	Ophiuroidea	Ophiuridae	Ophiuroidea OPNO 171b	East	9	2.149	0.849	0.065	0.450
44	Ophiuroidea	Ophiuridae	Ophiuroidea OPNO 171c	East	19	5.041	0.354	0.151	0.027*
45	Ophiuroidea	Ophiuridae	Ophiuroidea OPNO 177a	East	7	2.683	2.718	0.064	0.356
46	Ophiuroidea	Ophiuridae	Ophiuroidea OPNO 006	East	24	1.415	1.029	0.058	0.499
47	Echinoidea	Temnopleuridae	Temnopleuridae OPNO 142	West	18	1.742	0.320	0.023	0.970
48	Echinoidea	Temnopleuridae	Temnopleuridae OPNO 203a	East	18	0.684	60.401	0.128	0.079*
49	Echinoidea	Temnopleuridae	Temnopleuridae OPNO 203b	East	19	2.388	5.049	0.075	0.293
50	Echinoidea	Laganidae	Laganidae OPNO 142a	West	18	1.773	0.613	0.327	0.000*
51	Echinoidea	Laganidae	Laganidae OPNO 142a	East	12	6.750	0.048	0.051	0.610
52	Echinoidea	Laganidae	Laganidae OPNO 142b	West	6	5.018	0.259	0.035	0.868

Table 7. (continued).

Case	Class	Family	Scientific name	Region	N	shape $\hat{\nu}$	scale $\hat{\alpha}$	$\tilde{W}_N^2(\hat{\theta})$	p-value
53	Echinoidea	Brissidae	Brissidae OPNO 006	East	14	1.007	3.693	0.041	0.826
54	Crustacea	Penaeidae	*Metapenaeopsis novaeguineae*	West	11	1.728	0.194	0.110	0.097[*]
55	Crustacea	Penaeidae	*Parapenaeopsis cornuta*	West	14	1.213	0.582	0.128	0.070[*]
56	Crustacea	Penaeidae	*Parapenaeopsis tenella*	West	8	4.142	0.066	0.030	0.899
57	Crustacea	Diogenidae	*Dardanus imbricatus*	West	17	1.382	1.510	0.100	0.162
58	Crustacea	Diogenidae	*Dardanus imbricatus*	East	12	4.953	0.479	0.058	0.421
59	Crustacea	Paguridae	Paguridae OPNO 142	West	11	3.405	0.126	0.053	0.559
60	Crustacea	Dorippidae	Dorippe sp OPNO 142a	West	9	0.528	6.585	0.053	0.544
61	Crustacea	Leucosiidae	*Leucosia whitei*	East	8	27.462	0.049	0.071	0.293
62	Crustacea	Leucosiidae	*Leucosia ocellata*	East	17	69.968	0.038	0.130	0.056[*]
63	Crustacea	Leucosiidae	Leucosia sp OPNO 142	West	13	2.144	2.001	0.082	0.285
64	Crustacea	Leucosiidae	Arcania sp OPNO 008	East	16	6.768	0.050	0.136	0.052[*]
65	Crustacea	Matutidae	*Matuta inermis*	West	11	1.851	4.211	0.171	0.009[*]
66	Crustacea	Matutidae	*Matuta granulosa*	West	14	0.743	26.130	0.184	0.012[*]
67	Crustacea	Majidae	Hyastenus sp OPNO 214	East	9	1.760	1.748	0.037	0.767
68	Crustacea	Majidae	Hyastenus sp OPNO 054	West	6	10.607	0.031	0.027	0.924
69	Crustacea	Majidae	Majidae OPNO 154b	West	8	4.163	0.044	0.040	0.721
70	Crustacea	Parthenopidae	*Aulacolambrus hoplonotus*	East	14	1.136	2.426	0.050	0.576
71	Crustacea	Parthenopidae	*Parthenope nodosus*	West	16	1.414	2.895	0.064	0.425
72	Crustacea	Parthenopidae	*Parthenope longispinus*	East	7	6.572	0.816	0.091	0.159
73	Crustacea	Parthenopidae	Parthenope sp OPNO 060	West	10	7.953	0.226	0.038	0.770
74	Crustacea	Portunidae	*Portunus (Portunus) pelagicus*	West	8	0.229	95.524	0.266	0.002[*]
75	Crustacea	Portunidae	*Portunus (Monomia) rubromarginatus*	West	7	0.690	3.327	0.091	0.172
76	Crustacea	Portunidae	*Portunus (Xiphonectes) hastatoides*	East	7	22.983	0.027	0.036	0.781
77	Crustacea	Pilumnidae	*Pilumnus pugilator*	East	9	5.297	0.204	0.045	0.648
78	Ascidiacea	Clavelinidae	Clavelina sp OPNO 142	West	16	0.443	3.419	0.049	0.699
79	Ascidiacea	Ascidacea	Ascidian OPNO 211	East	11	2.655	2.501	0.058	0.508
80	Ascidiacea	Diazonidae	*Rhopalaea crassa*	East	9	2.825	9.045	0.117	0.060[*]

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